Reg. No. | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

UG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.
(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: B.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
III	PART - III	CORE - 5	U23MA305	VECTOR CALCULUS AND ITS APPLICATIONS

Date & Session: 24.04.2025/AN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** Answer ALL Questions.
CO1	K1	1.	If u(t) is a vector which is constant in magnitude, choose the value for $\frac{du}{dt}$. a) perpendicular to u b) equals to u c) parallel to u d) not equal to u.
CO1	K2	2.	The vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff select the correct value for $[\vec{a}\ \vec{b}\ \vec{c}]$ a) 1 b) $-[\vec{a}\ \vec{b}\ \vec{c}]$ c) 0 d) -1
CO2	K1	3.	If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then examine $\nabla . \vec{r}$ a) 3 b) 2 c) 1 d) 0
CO2	K2	4.	What is the value of $\nabla^2\varphi$, if $\nabla\varphi$ is solenoidal? a) 1 b) 0 c) 2 d) 4
CO3	K1	5.	Identify the parametric equation of the line joining (0, 0, 0) to (2, 1, 1). a) $x = y = t^2, z = t^3, 0 \le t \le 2$ b) $x = t^2, y = z = t, 0 \le t \le 1$ c) $x = z = t, y = t^2, 0 \le t \le 1$ d) $x = 2t, y = z = t, 0 \le t \le 1$.
CO3	K2	6.	Enumerate $\int_C \vec{r} . dr$, where C is the straight line joining (0, 0, 0) and (1, 1, 1) a) 0 b) 1 c) $\frac{3}{2}$ d) $\frac{2}{3}$
CO4	K1	7.	What is the value of $\int_0^1 \int_0^1 (x^2 + y^2)\, dx\, dy$? a) 0 b) 1 c) $\frac{1}{3}$ d) $\frac{2}{3}$
CO4	K2	8.	Select the correct value for $\int_0^a \int_0^a \int_0^a dz\, dy\, dx$. a) 0 b) a c) a^2 d) a^3.
CO5	K1	9.	If R is any closed region of the xy plane bounded by a simple closed curve C, what is the value of $\int_C y\, dx + x\, dy$ a) 0 b) 1 c) π d) 2π.
CO5	K2	10.	Which option is suitable for Green's theorem? a) Line integral and surface integral b) surface integral and volume integral c) double integral and surface integral d) Line integral and double integral.

Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** Answer ALL Questions choosing either (a) or (b)
CO1	K3	11a.	Compute $\frac{d}{dt}[\vec{f}\ \vec{g}\ \vec{h}] = \left[\vec{f}\ \vec{g}\ \frac{d\vec{h}}{dt}\right] + \left[\vec{f}\ \frac{d\vec{g}}{dt}\ \vec{h}\right] + \left[\frac{d\vec{f}}{dt}\ \vec{g}\ \vec{h}\right]$ **(OR)**

CO1	K3	11b.	If $\vec{f} = 5t^2\vec{i} + t\vec{j} - t^3\vec{k}$ and $\vec{g} = \sin t\,\vec{i} - \cos t\,\vec{j}$, find i) $\frac{d}{dt}(\vec{f}\cdot\vec{g})$ ii) $\frac{d}{dt}(\vec{f}\times\vec{g})$ iii) $\frac{d}{dt}(\vec{f}\cdot\vec{f})$
CO2	K3	12a.	If $\nabla\varphi = 2xyz^3\,\vec{i} + x^2z^3\,\vec{j} + 3x^2yz^2\,\vec{k}$, then find $\varphi(x,y,z)$ if $\varphi(1,-2,2) = 4$. **(OR)**
CO2	K3	12b.	Determine $curl\,(\vec{f}\times\vec{g}) = (\vec{g}\cdot\nabla)\vec{f} - (\vec{f}\cdot\nabla)\,\vec{g} + \vec{f}\,div\,\vec{g} - \vec{g}\,div\,\vec{f}$.
CO3	K4	13a.	Examine $\int_C \vec{f}\cdot d\vec{r}$, where $\vec{f} = (x^2 + y^2)\,\vec{i} + (x^2 - y^2)\,\vec{j}$ and C is the curve $y = x^2$ joining (0, 0) and (1, 1). **(OR)**
CO3	K4	13b.	Inspect the work done by the force $\vec{F} = 3xy\,\vec{i} - 5z\,\vec{j} + 10x\,\vec{k}$ along the curve C, $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from t = 1 to t = 2.
CO4	K4	14a.	Examine $\iint_S \vec{f}\cdot n\,dS$, where $\vec{f} = (x + y^2)\,\vec{i} - 2x\,\vec{j} + 2yz\,\vec{k}$ and S is the surface of the plane 2x + y + 2z = 6 in the first octant. **(OR)**
CO4	K4	14b.	Using triple integral discover the volume of the sphere $x^2 + y^2 + z^2 = a^2$.
CO5	K5	15a.	Justify Green's theorem for the function $\vec{f} = (x^2 + y^2)\,\vec{i} - 2xy\,\vec{j}$ and C is the rectangle in the xy plane bounded by y = 0, y = b, x = 0 and x = a. **(OR)**
CO5	K5	15b.	Prove that for a closed surface S, $\iint_S \vec{r}\cdot n\,dS = 3V$, where V is the volume enclosed by S.

Course Outcome	Bloom's K-level	Q. No.	**<u>SECTION – C</u> (5 X 8 = 40 Marks)** **Answer <u>ALL</u> Questions choosing either (a) or (b)**
CO1	K3	16a.	Manipulate $\frac{d}{dt}(u\times v) = u\times\frac{dv}{dt} + \frac{du}{dt}\times v$ **(OR)**
CO1	K3	16b.	If $r = a\cos\omega t + b\sin\omega t$, where a, b are constant vectors and ω is a constant, determine the followings: $r\times\frac{dr}{dt} = \omega\,(a\times b)$ and $\frac{d^2r}{dt^2} + \omega^2 r = 0$
CO2	K4	17a.	Construct the equation of the (i) tangent line and (ii) normal plane to the curve of intersection of the surfaces $3x^2 + y^2z + 2 = 0$; $2xz - x^2y - 3 = 0$ at the point (1, -1, 1) **(OR)**
CO2	K4	17b.	Inspect $div\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$.
CO3	K4	18a.	Examine $\int_C \vec{f}\cdot d\vec{r}$, where $\vec{f} = (x^2 + y^2)\,\vec{i} - 2xy\,\vec{j}$ and the curve C is the rectangle in the xy plane bounded by y = 0, y = b, x = 0, x = a. **(OR)**
CO3	K4	18b.	If $\vec{f} = (2y + 3)\,\vec{i} + xz\,\vec{j} + (yz - x)\,\vec{k}$, examine $\int_C \vec{f}\cdot d\vec{r}$ along the following paths C (i) $x = 2t^2$, $y = t$, $z = t^3$ from t = 0 to t = 1. (ii) the polygonal path P consisting of the three-line segments AB, BC and CD where A = (0, 0, 0), B = (0, 0, 1), C = (0, 1, 1) and D = (2, 1, 1). (iii) the straight line joining (0, 0, 0) and (2, 1, 1).
CO4	K5	19a.	Evaluate $\iint_S \vec{f}\cdot n\,dS$, where $\vec{f} = (x^3 - yz)\,\vec{i} - 2x^2y\,\vec{j} + 2\,\vec{k}$ and S is the surface of the cube bounded by x = 0, y = 0, z = 0, x = a, y = a and z = a. **(OR)**
CO4	K5	19b.	Evaluate the volume of the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.
CO5	K5	20a.	Justify Gauss divergence theorem for $\vec{f} = y\,\vec{i} + x\,\vec{j} + z^2\,\vec{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2$; z = 0 and z = h. **(OR)**
CO5	K5	20b.	Justify Stokes theorem for the vector function $\vec{f} = y^2\,\vec{i} + y\,\vec{j} - xz\,\vec{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$; $z \geq 0$.